

Applications of linear equations

“When am I ever going to use this?” “Where would this be applicable?” All the way through math, students ask questions like these. Well, to the relief of some and the dismay of others, you have now reached the point where you will be able to do some problems that have been made out of real life situations. Most commonly, these are called, “story problems”.

The four main points to remember are:

D- Data. Write down all the numbers that may be helpful. Also, note any other clues that may help you unravel the problem.

V- Variable. In all of these story problems, there is something that you don’t know, that you would like to. Pick any letter of the alphabet to represent this.

P- Plan. Story problems follow patterns. Knowing what kind of problem it is, helps you write down the equation. This section of the book is divided up so as to explain most of the different kinds of patterns.

E- Equation. Once you know how the data and variable fit together. Write an equation of what you know. Then solve it. This turns out to be the easy part.

Once you have mastered the techniques in solving linear equations, then the fun begins. Linear equations are found throughout mathematics and the real world. Here is a small outline of some applications of linear equations. You will be able to solve any of these problems by the same methods that you have just mastered.

Translation

The first application is when you simply translate from English into math. For example:
Seven less than 3 times what number is 39?

Since we don’t know what the number is, we pick a letter to represent it (you can pick what you would like to); I will pick the letter x:

$$\begin{aligned} 3x - 7 &= 39 \text{ then solve} \\ 3x &= 46 \\ x &= \frac{46}{3} \quad (\text{or } 15\frac{1}{3} \text{ or } 15.\bar{3}) \end{aligned}$$

That’s the number.

Substitution

Sometimes you are given a couple of different things to find. Example:

Two numbers add to 15, and the second is 7 bigger than the first. What are the two numbers?

Pick some letters to represent what you don't know. Pick whatever is best for you. I will choose the letter "f" for the first number and "s" for the second. I then have two equations to work with:

$$\begin{aligned} f + s &= 15 \quad \text{and} \quad s = f + 7 \\ f + f + 7 &= 15 \\ 2f + 7 &= 15 \\ 2f &= 8 \\ f &= 4 \end{aligned}$$

The letter "s" and "f+7" are exactly the same and can be changed places.

4 must be the first number, but we need to stick it back in to one of the original equations to find out what "s" is.

$$\begin{aligned} s &= f + 7 \\ &= 4 + 7 \\ &= 11. \quad 4 \text{ and } 11 \text{ are our two numbers.} \end{aligned}$$

These kind of problems often take the form of an object being cut into two pieces. Here, I will show you what I mean.

Example:

A man cuts a 65 inch board so that one piece is four times bigger than the other. What are the lengths of the two pieces?

Now, I would personally pick "f" for first and "s" for second. We know that

$$f + s = 65 \quad \text{and that } s = 4f$$

Thus,

$$\begin{aligned} f + 4f &= 65 \\ 5f &= 65 \\ f &= 13, \text{ so the other piece must be } 52. \end{aligned}$$

The pieces are 13in and 52in.

Shapes

With many of the problems that you will have, pictures and shapes will play a very important role. When you encounter problems that use rectangles, triangles, circles or any other shape, I would suggest a few things:

1. Read the problem
2. READ the problem again.
3. READ THE PROBLEM one more time.

Once you draw a picture to model the problem – read the problem again to make sure that your picture fits.

The formulas for the shapes that we will be discussion are found in Section 2.2.

Variable on Both Sides

Unfortunately, not all equations come out such that this un-doing technique works. Sometimes the x shows up in several different places at once:

$$3x - 5 + 2x - 3 = 5x + 7(x - 8)$$

Seeing all of the x 's scattered throughout the equation sometimes looks daunting, but it isn't as bad as all that. We know a couple of ways to make it look a bit more simple.

$$\begin{array}{l}
 3x - 5 + 2x - 3 = 4x + 7(x - 8) \text{ becomes} \\
 \swarrow \quad \searrow \quad \swarrow \quad \searrow \\
 5x - 8 = 4x + 7x - 56 \quad \text{Distribute the 7 and combine} \\
 \swarrow \quad \searrow \\
 5x - 8 = 11x - 56 \quad \text{Combine the like terms}
 \end{array}$$

Now we reach a point where you should feel somewhat powerful. Remember that you can **add, subtract, multiply or divide** anything you want! (As long as you do it to both sides).

Particularly, I don't like the way that $11x$ is on the left hand side. I *choose* to get rid of it! So, I subtract $11x$ from both sides of the equation:

$$\begin{array}{r}
 5x - 8 = 11x - 56 \\
 -11x \quad -11x \\
 \hline
 \end{array}$$

Upon combining the like-terms, I get

$$-6x - 8 = -56$$

Which now is able to be un-done easily:

$$\begin{array}{l}
 -6x = -48 \quad (\text{add 8 to both sides}) \\
 x = 8 \quad (\text{divide both sides by } -6)
 \end{array}$$

Special cases: What about $2x + 1 = 2x + 1$

Well if we want to get the x 's together we had better get rid of the $2x$ on one side. So we subtract $2x$ from both sides like this:

$$\begin{array}{r}
 2x + 1 = 2x + 1 \\
 -2x \quad -2x \\
 \hline
 \end{array}$$

You might as well know that if you didn't like the $5x$ on the right hand side, you could get rid of that instead:

$$\begin{array}{r}
 5x - 8 = 11x - 56 \\
 -5x \quad -5x \\
 \hline
 \end{array}$$

Combining like terms, we get:

$$\begin{array}{r}
 -8 = 6x - 56 \\
 48 = 6x \quad (\text{add 56 to both sides}) \\
 8 = x \quad (\text{divide both sides by 6})
 \end{array}$$

We will always get the same answer!
You can't mess up!

$$0 = 0$$

$$5 = 5$$

$$-3 = -3$$

solution is **all real numbers**

$$1 = 1$$

Ahh! The x's all vanished.

Well, what do you think about that? This statement is always true no matter what x is. That is the point. x can be any number it wants to be and the statement will be true. **All numbers are solutions.**

$$0 = 1$$

$$5 = 7$$

$$-3 = 2$$

No solution

On the other hand try to solve:

$$2x + 1 = 2x - 5$$

$$-2x \quad -2x$$

$$1 = -5$$

Again, the x's all vanished. This time it left an equation that is never true. No matter what x we stick in, we will never get 1 to equal -5. It simply will never work. **No solution.**

Section 3.1 Exercises Part B

Simplify.

1. $4s(t-9) - t(s+11)$

2. $12(x^2-5n) + 3n - 4x^2$

3. $6nj - 7j + 8nj + 11n$

Solve.

4. $5\left(\frac{6x-4}{5} + 2\right) = 30$

5. $7\left(\frac{-2x+8}{6} + 5\right) - 2 = 12$

6. $-3 - 7m = 18$

7. $\frac{7}{2}t = -14$

8. $-15 = 3x + 9$

9. $\frac{2x-7}{3} = 33$

10. $t + 5t + 4t - 7 = 17$

11. $9\left(\frac{5x-8}{6} + 7\right) - 3 = 42$

Solve for the specified variable.

12. $y = mx + b$ for x

13. $\frac{5m+9}{2} = r$ for m

14. $6 = 7b - pb$ for b

15. $3t + nt = y$ for t

16. $P = 2l + 2w$ for l

17. $SA = 2\pi rh + 2\pi r^2$ for h

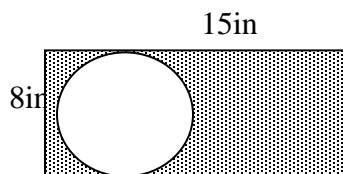
18. 27 is 6 more than 3 times a number. What is the number?

19. 18 less than 5 times a number is 52. What is the number?

20. Two numbers add to 37 and the second is 9 bigger than the first. What are the two numbers?

21. Two numbers add to 238 and the first is 34 bigger than the second. What are the two numbers?

22. Find the area of the shaded region:



23. I have created a triangular garden such that the largest side is 6ft less than twice the smallest and the medium side is 5ft larger than the smallest side. If the total perimeter of the garden is 47ft, what are the lengths of the three sides?

24. If a rectangle's length is 5 more than twice the width and the perimeter is 46 mm, what are the dimensions for the rectangle?

25. If a cone has a Lateral Surface Area of 250 in^2 , a radius of 8in, what is the slant height of the cone? Use a calculator.

26. Two numbers add to 589 and the first is 193 bigger than the second. What are the two numbers?

27. If a cylinder has a volume of 538 cm^3 and a radius of 6 cm, how tall is it?

28. Find the missing variable for a rectangle:

$$\begin{aligned} P &= 39 \text{ ft} \\ w &= 7.2 \text{ ft} \\ l &= \end{aligned}$$

29. Find the missing variable for a cylinder:

$$\begin{aligned} SA &= 800 \text{ in}^2 \\ h &= \\ r &= 9 \text{ in} \end{aligned}$$

Solve.

Example:

$x + 4 - 5x = 7x + 1$	Combine like terms
$-4x + 4 = 7x + 1$ $+4x \quad +4x$	Get all x's together by adding 4x to both sides
$4 = 11x + 1$ $-1 \quad -1$	Subtract 1 from both sides
$3 = 11x$	
$\frac{3}{11} = x$	Divide both sides by 11

30. $4p + 2 = 7p - 6$

31. $-4n + 5 = n$

32. $2x - 7 = x + 5$

33. $x - 42 = 15x$

34. $-4(x-3) = -2x + 12$

35. $7x = 13 + 7x$

36. $.4x - 1 = .9x + 5$

37. $2(x - 4) = 3x - 14$

38. $.4y + 78 = 78 + .4y$

Answers:

1. $3st - 36s - 11t$
2. $8x^2 - 57n$
3. $14nj - 7j + 11n$
4. $x = 4$
5. $x = 13$
6. $m = -3$
7. $t = -4$
8. $x = -8$
9. $x = 53$
10. $t = \frac{12}{5}$ or 2.4
11. $x = -\frac{4}{5}$
12. $x = \frac{y-b}{m}$
13. $m = \frac{2r-9}{5}$
14. $b = \frac{6}{7-p}$
15. $t = \frac{y}{3+n}$
16. $l = \frac{p-2w}{2}$
17. $h = \frac{SA - 2\pi r^2}{2\pi r}$
18. 7
19. 14
20. 14, 23
21. 102, 136
22. 69.73 in^2
23. 12, 17, 18
24. $l = 17\text{mm}; w = 6\text{mm}$
25. $\ell = 9.95 \text{ in}$
26. 198, 391
27. 4.76 cm
28. $l = 12.3 \text{ ft}$
29. $h = 5.15 \text{ in}$
30. $p = \frac{8}{3}$ or 2.67
31. $n = 1$
32. $x = 12$
33. $x = -3$
34. $x = 0$
35. No solution
36. $x = -12$
37. $x = 6$
38. All numbers