

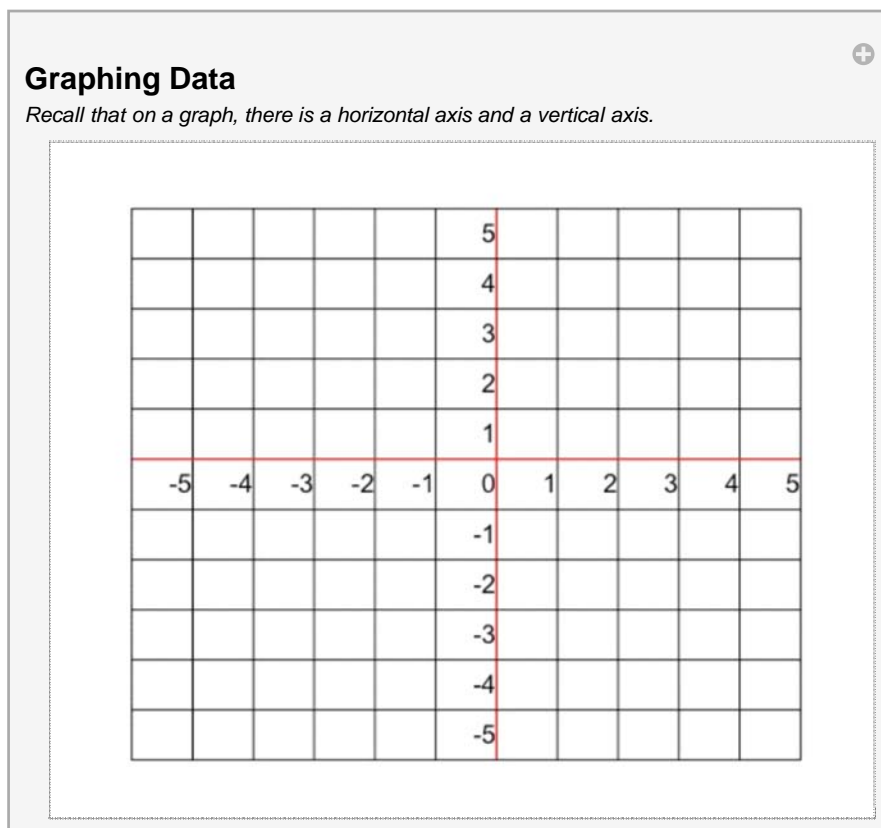
# Math Review

**Acknowledgement:** BYU-Idaho Economics Department Faculty (Principal authors: Rick Hirschi, Ryan Johnson, Allan Walburger and David Barrus)

## Section 1 - Graphing Data

### Graphs

It is said that a picture is worth a thousand words. Likewise, a **graph** can often be used to better understand relationships between variables. Recall that on a graph, there is a horizontal axis and a vertical axis.



Along the **horizontal axis**, we start at the origin or a zero. To the right numbers increase and to the left numbers decrease.

## Graphing Data – Horizontal Axis

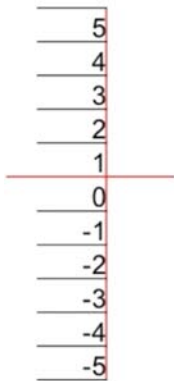
Along the horizontal axis, we start at the origin or at zero. To the right numbers increase, and to the left numbers decrease.



On the **vertical axis**, we also start at the origin or zero and the numbers as we go up increase and the numbers as we go down decrease.

## Graphing Data – Vertical Axis

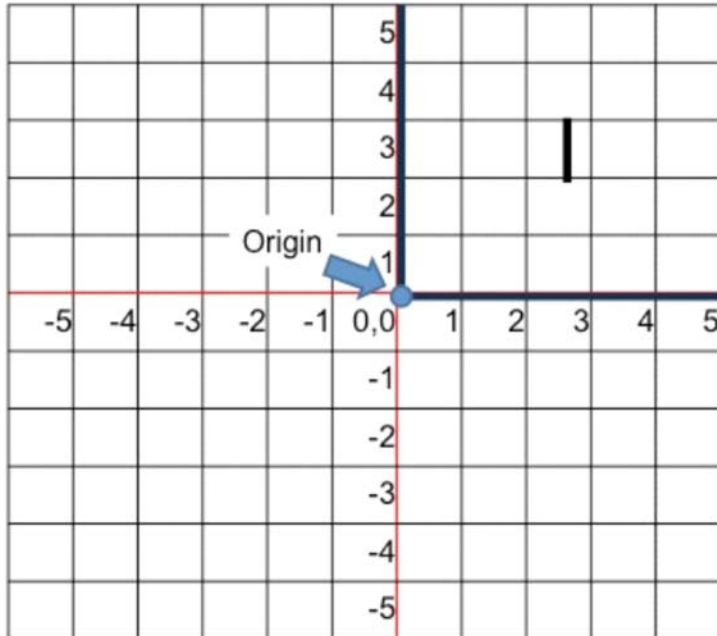
On the vertical axis, we also start at the origin or zero. The numbers increase as we go up, and the numbers decrease as we go down.



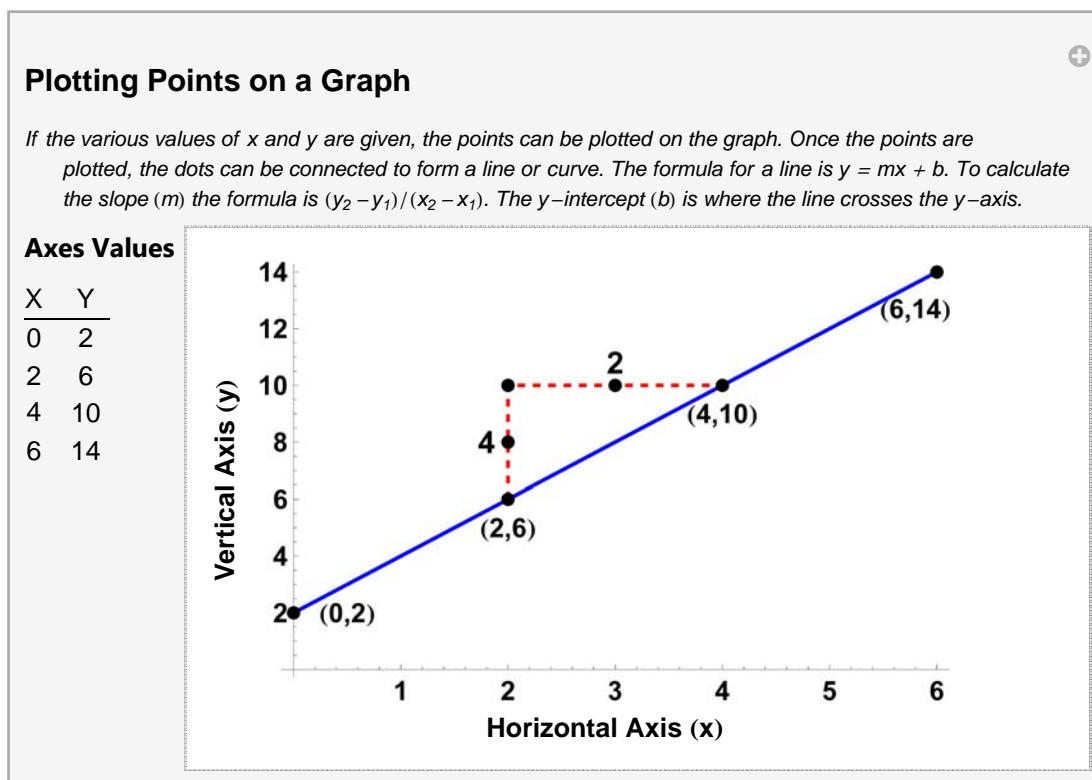
Combining the horizontal and vertical axes allow us to look at the relationships between variables in two-dimensional space. While we will at times use all four quadrants, we typically spend most of our time in the first quadrant or the space where both the horizontal and vertical variables are positive.

## Graphing Data

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## Plotting Points on a Graph



If the various **values of  $x$  and  $y$**  are given, the points can be plotted on the graph.

Once the points are plotted, the dots can be connected to form a line or curve. In our equation, there is a positive relationship between  $x$  and  $y$ . When  $x$  increases by one,  $y$  increases by 2. The slope of a line is determined by taking the change in the vertical amount divided by the change in the horizontal amount. We will let the Greek symbol Delta ( $\Delta$ ) represent the change. The formula to calculate the slope of the curve is  $\text{rise/run} = (\Delta y) / (\Delta x) = (y_2 - y_1) / (x_2 - x_1)$ . In our example, as  $x$  increases by 2,  $y$  increases by 4 so the slope would be a positive 2. The  $y$ -intercept is where the line crosses the  $y$ -axis. In this example, the  $y$ -intercept is 2. The generic formula for a line is  $y = mx + b$ , where  $m$  is the slope, and  $b$  is the  $y$ -intercept. The equation for the line above is  $y = (4/2)x + 2$ . We can simplify it to  $y = 2x + 2$ .

In our example, there is a **linear relationship** between  $x$  and  $y$ . We can express the same information, using an equation. The intercept value is the value of  $y$  when  $x$  is zero.

In modeling causal relationships, the value of an **endogenous** or **dependent variable** depends upon the value of other variables in the model. When graphing, we typically place the dependent variable on the vertical or  $y$ -axis. Those variables whose values are determined outside the model are known as **exogenous** or **independent variables**. These are typically graphed on the  $x$  or horizontal axis.

A **positive** or **direct relationship** between the two variables occurs when the increase in the independent variable causes an increase in the dependent variable. For example, as income increases the amount people will consume also increases.

A **negative** or **inverse relationship** occurs when the increase in one variable decreases the value of the other variable. As the price of a good falls, the number of people willing to buy the good increases.

From an equation, we are able to derive the points in a table by plugging in the values of  $x$  and computing the value of  $y$ .

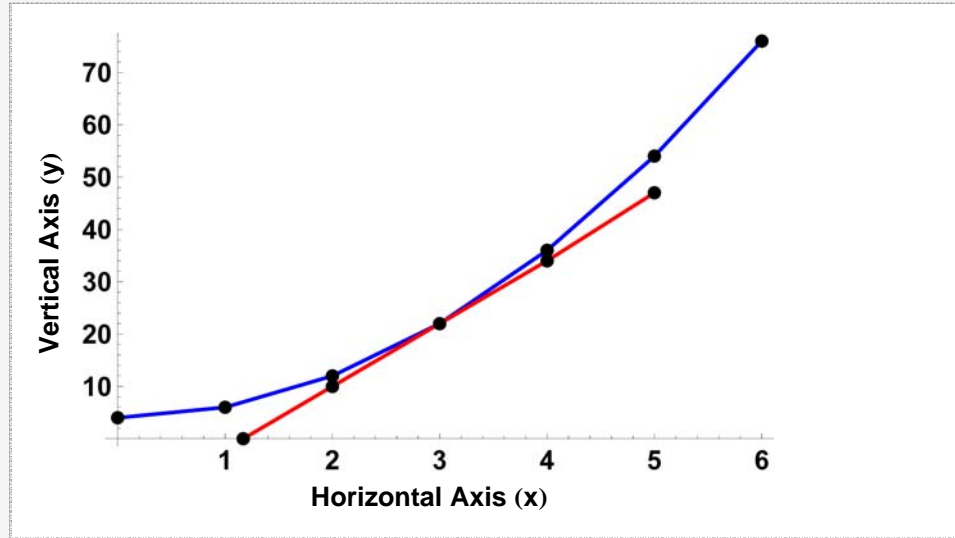
## Slope of a Curve

### Slope of a Curve with Tangent

A curve will have different slopes at different points along the curve. By taking the change in  $y$  over the change in  $x$ , we are able to get an estimate of the curve at that point. As shown in the table, the slope gets steeper as we move along the curve.

#### Axes and Slope Values

| X | Y  | Slope |
|---|----|-------|
| 0 | 4  |       |
| 1 | 6  | 2     |
| 2 | 12 | 6     |
| 3 | 22 | 10    |
| 4 | 36 | 14    |
| 5 | 54 | 18    |
| 6 | 76 | 22    |



A curve will have different slopes at different points along the curve. By taking the change in  $y$  over the change in  $x$ , we are able to get an estimate of the curve at that point. As shown in the table, the slope gets steeper as we move along the curve. When  $x = 1$  and  $y = 6$  the slope is 2, but when  $x = 6$  and  $y = 76$  the slope is 22.

If we compute the slope over a distance, we find that the slope from  $x = 0$  to  $x = 6$  is 12 and from  $x = 2$  to  $x = 4$  is also 12. If we were to draw a **tangent line** (a line that touches the curve only at one place) where  $x = 3$  we could then compute the slope of the tangent. In doing so, we would find the slope is again 12. The graph above shows the line tangent to the curve when  $x = 3$ .

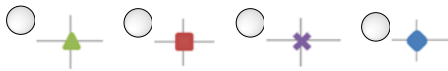
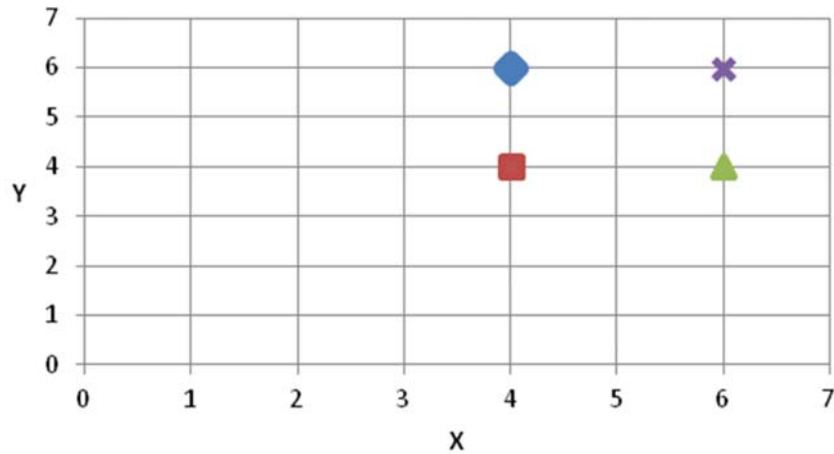
## Ponder and Prove - Section 1 - Graphing Data

### Section 1 Questions

Instructions: Click on the button that represents the correct answer. After you select an answer, click on the 'Grade My Answer' button.

Question 1 Question 2 Question 3 Question 4 Question 5

Which of these points depicts the point where  $X=6$  and  $Y=4$ ?



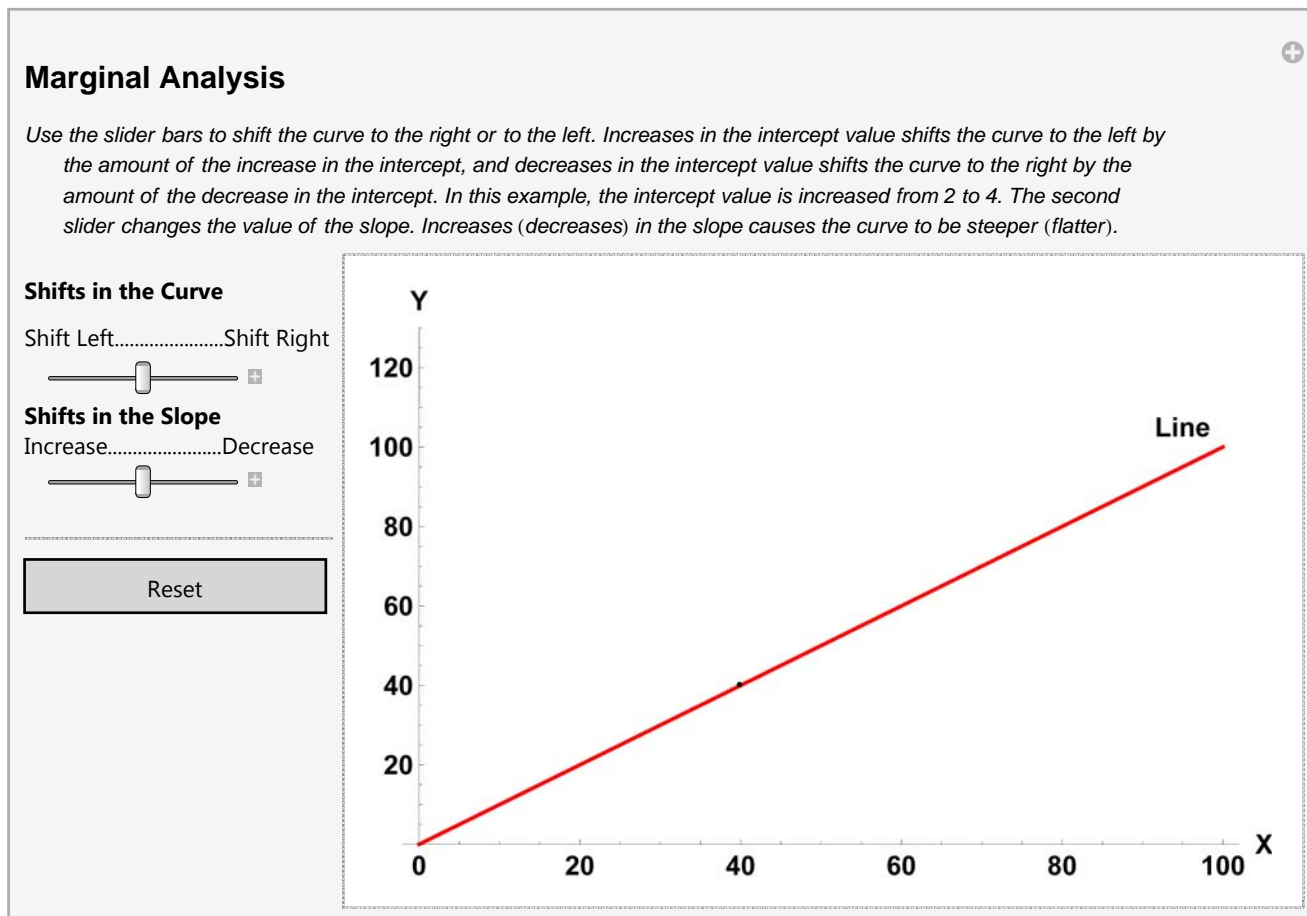
Grade My Answer

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## Section 2 - Marginal Analysis and Solving for Equilibrium

### Marginal Analysis



One of the key tools in economics is marginal analysis. As economists look at relationships between variables, they want to know how a change in one variable is correlated with a change in another variable. As the change in  $x$  gets smaller and smaller, we can find the slope of the tangent line or the slope of the line at one point along the curve. Thus, being able to compute the slope of the tangent tells us what the marginal or additional change is in the dependent variable. Or in other words, if  $x$  changes by a little bit we can see how much  $y$  is going to change.

#### Slope of the tangent = $\Delta Y / \Delta X$ as the change in $x$ gets very small

One powerful mathematical tool in calculus is called the derivative. The derivative tells us the slope of the curve at a particular point. Although it is not a requirement for this class, an example is provided for those who have previously had calculus. The derivative simply tells us the change in  $y$  given a very small change in  $x$ , or in other words the slope of the curve at a particular point. Given the function,  $Y = 4 + 2x^2$ , the first derivative gives us a slope of the tangent at a given point. To find the slope of the tangent when  $x = 3$ , we plug in the value of 3 for  $x$  in the first derivative and find the slope is 12.

**The derivative gives us the slope of the curve.**

**$\Delta Y / \Delta X = \text{slope of the curve}$**

$$Y = 4 + 2x^2$$

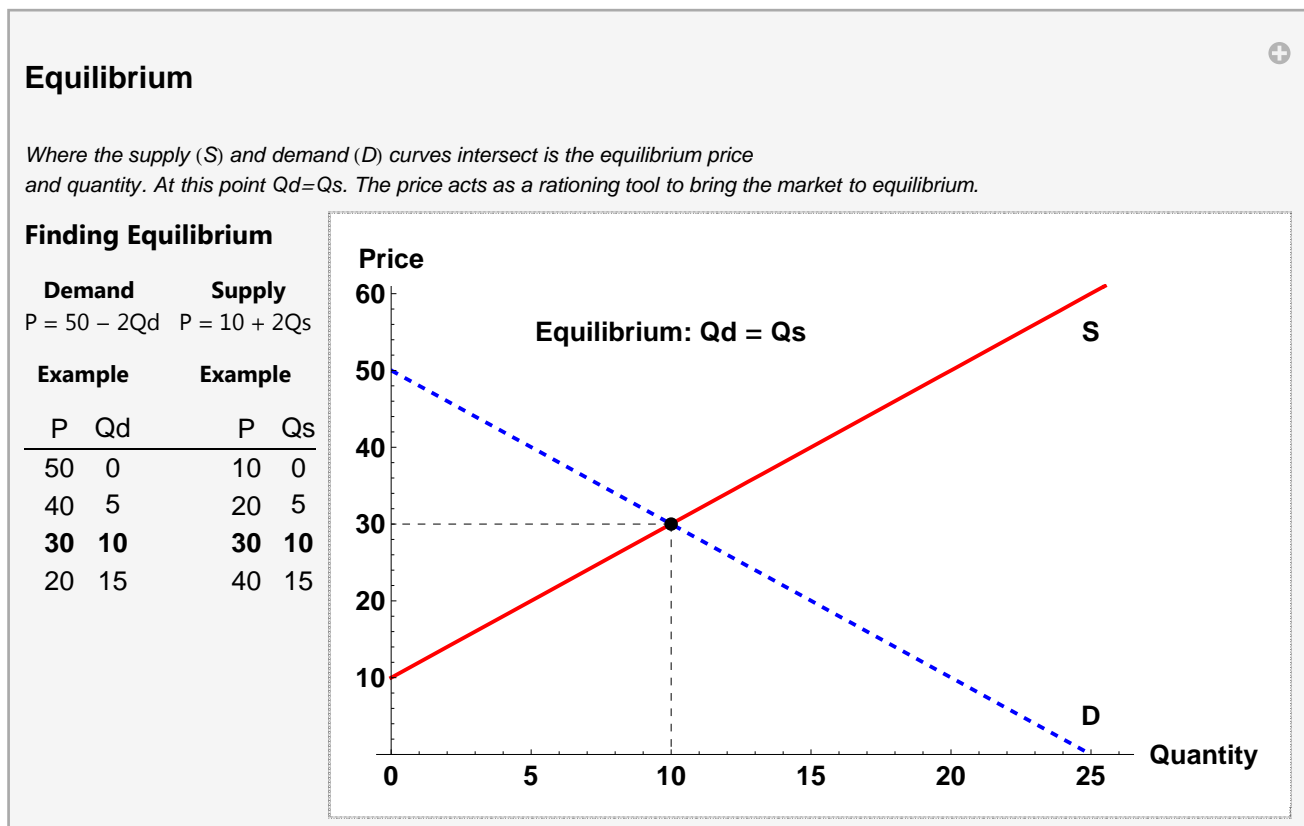
$$\text{First derivative} = \Delta Y / \Delta X = 4x$$

$$\text{Slope of the tangent when } x = 3 \text{ is } 4(3) = 12$$

Increases in the intercept value shifts the curve upwards by the amount the value of the intercept changes. In our example, the intercept value is increased from 2 to 4.

Increases (decreases) in the slope causes the curve to be steeper (flatter).

## Equilibrium



Often, we want to find an **equilibrium** where the one curve would intersect with another, such as with supply and demand.

We are able to find the market equilibrium either by graphing or algebraically. First we need to define what some variables mean.  $Q_s$  = quantity supplied.  $Q_d$  = quantity demanded.  $P$  = price. If on the demand side, we are given  $P = 50 - 2Q_d$  and on the supply side, we are given  $P = 10 + 2Q_s$ . We could simply plug in values, plot the points, and find where demand intersects supply.

Even without graphing the curves, we are able to see that at a price of \$30, the quantity demanded equals the quantity supplied.

If we graph it, we find that at price of 30 dollars, the quantity supplied would be 10 and the quantity demanded would be 10.

## Solving Equilibrium Algebraically



An alternative way, is to solve the problem algebraically. Since at equilibrium the quantity supplied equals the quantity demanded and the price will be the same for both, we are able to set the two equations equal and solve.

Our first step is to get the Q together, by adding 2Q to both sides. On the left hand side, the negative 2Q plus 2Q cancel each other out, and on the right side 2 Q plus 2Q gives us 4Q.

Our next step is to get the Q by itself. We can subtract 10 from both sides and are left with  $40 = 4Q$ . The last step is to divide both sides by 4, which leaves us with an equilibrium Quantity of 10.

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### Step-by-Step Solving Algebraically

|   |  |          |
|---|--|----------|
| 1. Quantity supplied equals quantity demanded at equilibrium:<br>(step 1)   | $Q_d = Q_s = Q$  |          |
| 2. Set the two equations equal to each other:                               | $50 - 2Q = 10 + 2Q$  | (step 2) |
| 3. Get Qs together. Add 2Q to both sides:<br>(step 3)                       | $50 - 2Q + 2Q = 10 + 2Q + 2Q$<br>$50 = 10 + 4Q$                  |          |
| 4. Get Q by itself by subtracting 10 from both sides:<br>(step 4)           | $50 - 10 = 10 - 10 + 4Q$<br>$40 = 4Q$                            |          |
| 5. Divide both sides by 4. Solve for eq. quantity = Q*:<br>(step 5)         | $40/4 = 4Q/4$<br>$10 = Q^*$                                      |          |
| 6. Plug $Q^* = 10$ back into an equation to get eq. price = P*:<br>(step 6) | $P = 50 - 2(10)$ or $P = 10 + 2(10)$<br>$P^* = 30$ or $P^* = 30$ |          |

## Ponder and Prove - Section 2 - Marginal Analysis and Solving for Equilibrium

## Section 2 Questions

Instructions: Click on the button that represents the correct answer. After you select an answer, click on the 'Grade My Answer' button.

Question 1 Question 2 Question 3 Question 4

Going from equation  $Y=2+7X$  to  $Y=5+7X$  causes the line to:

- Shift Right  
 Get Flatter  
 Shift Left  
 Get Steeper

Grade My Answer

Reset

"Results"

## Section 3: Optimums, Charts & Graphs, and Types of Data

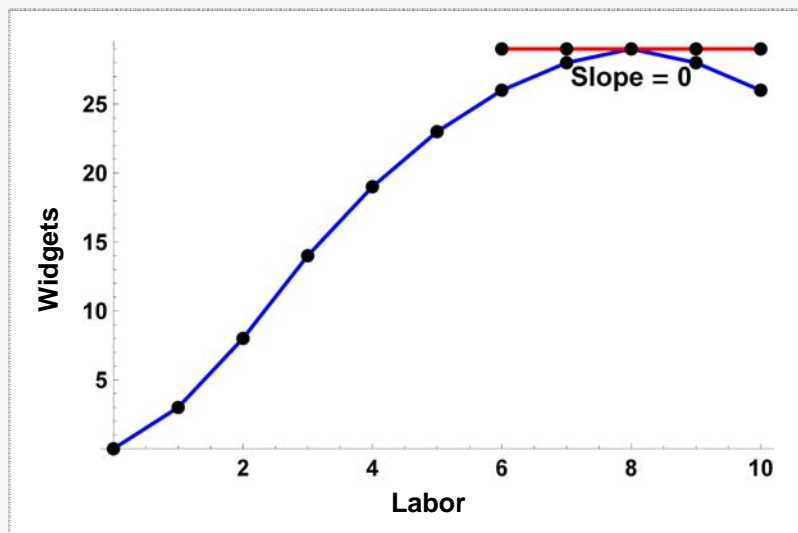
### Determining the Optimums

#### Determining the Optimums

Finding the slope of a curve can be extremely useful when determining the maximum or minimum. For example, let's say a company uses labor to produce a product, called widgets. The additional or marginal output from each additional worker initially increases, then decreases as the other resources become relatively more scarce.

#### Axes and Slope Values

| Labor | Output | Marginal Output |
|-------|--------|-----------------|
| 0     | 0      |                 |
| 1     | 3      | 3               |
| 2     | 8      | 5               |
| 3     | 14     | 6               |
| 4     | 19     | 5               |
| 5     | 23     | 4               |
| 6     | 26     | 3               |
| 7     | 28     | 2               |
| 8     | 29     | 1               |
| 9     | 28     | -1              |
| 10    | 26     | -2              |



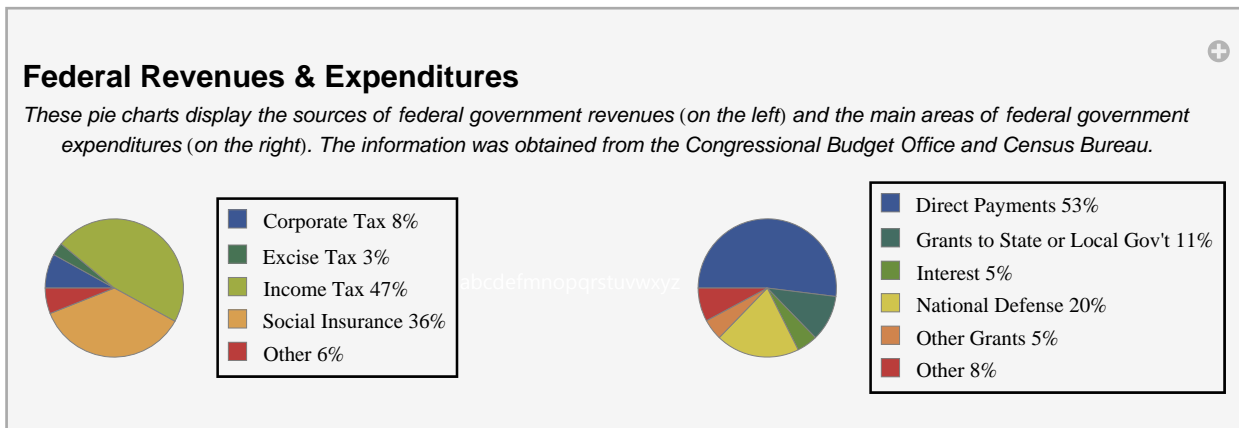
Math as a tool enables economists to determine the optimums. For example, where profits are maximized or costs are minimized.

Finding the slope of a curve can be extremely useful when determining the **maximum** or **minimum**. For example, let's say a company uses labor to produce a product, called widgets. The additional or marginal output from each additional worker initially increases, then decreases as the other resources become relatively more scarce.

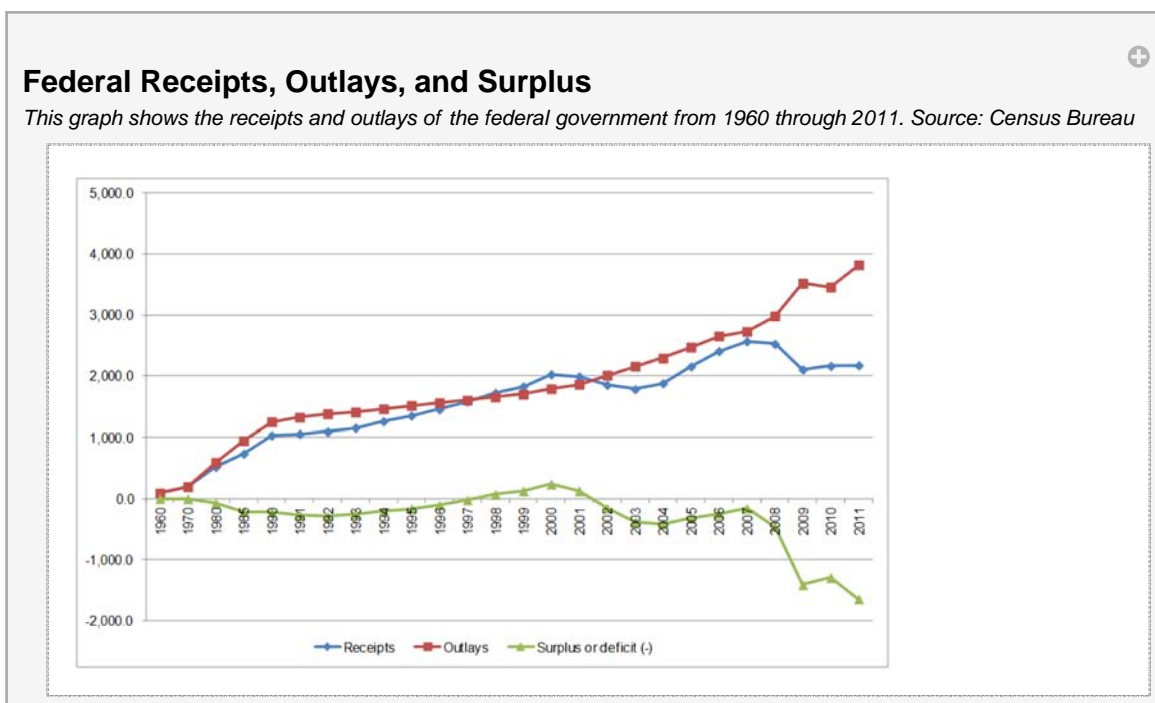
If the company wants to find how many workers will maximize their output, they would look at the point where the slope of the curve goes to zero. In our example if the company wanted to maximize output, it would use 8 workers to do so (see graph above).

## Charts and Graphs

Charts and graphs are frequently used to display information. A **pie chart** shows the portion or relative magnitude of the total amount that is made up by each of the components. For example, these pie charts display the sources of federal government revenues and the main areas of federal government expenditures.

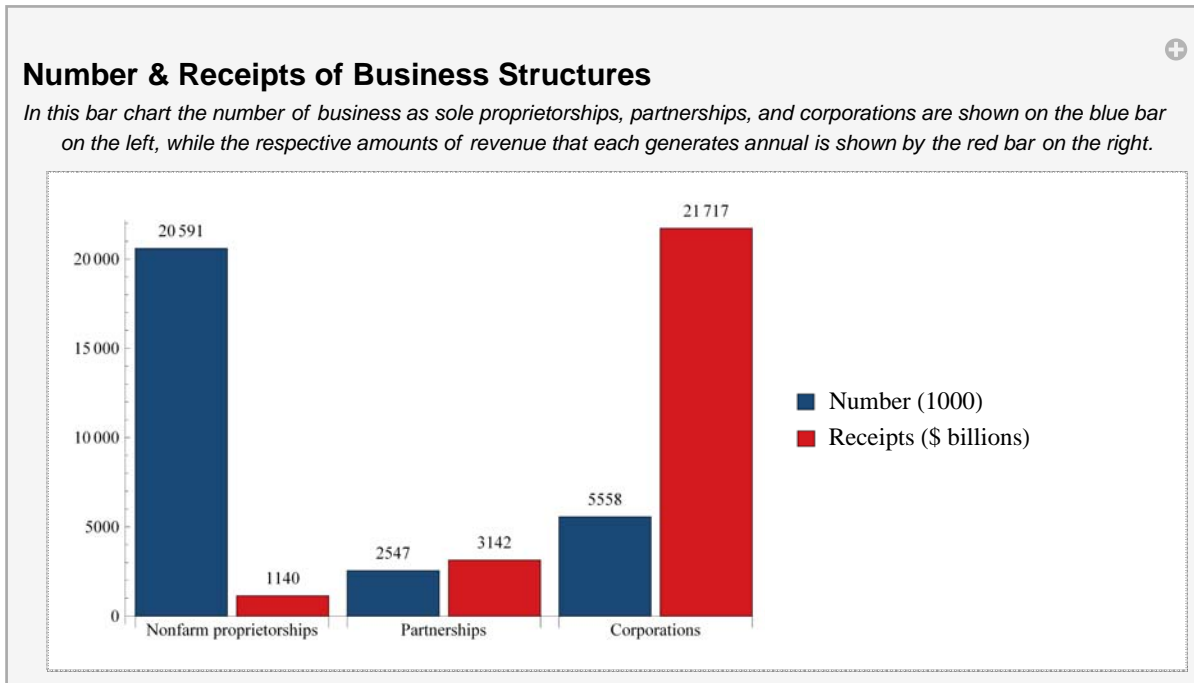


A **time series** displays the data values over time. Time is along the horizontal axis and the observations are evenly spaced, such as annually, quarterly, or monthly. This graph below shows the receipts and outlays of the federal government and the budget surplus on the primary or vertical axis.



A **bar chart** shows the amount or magnitude of each item by the length of the item. In the bar chart below the num-

ber of business as sole proprietorships, partnerships, and corporations are shown on the left (blue) while the respective amounts of revenue that each generates annual is shown by the bar on the right (red).



## Types of Data

**Cross sectional data** looks at various segments of a population at a point in time. This table below displays the median weekly earnings and the unemployment rate in 2007 based on the level of education. As seen from the data, those with more education tend to have greater earnings and lower unemployment rates.

## Cross Section Data

This table displays the median weekly earnings and the unemployment rate in 2007 based on the level of education. As seen from the data, those with more education tend to have greater earnings and lower unemployment rates. Source: Bureau of Labor Statistics, <http://www.bls.gov/emp/emptab7.htm>

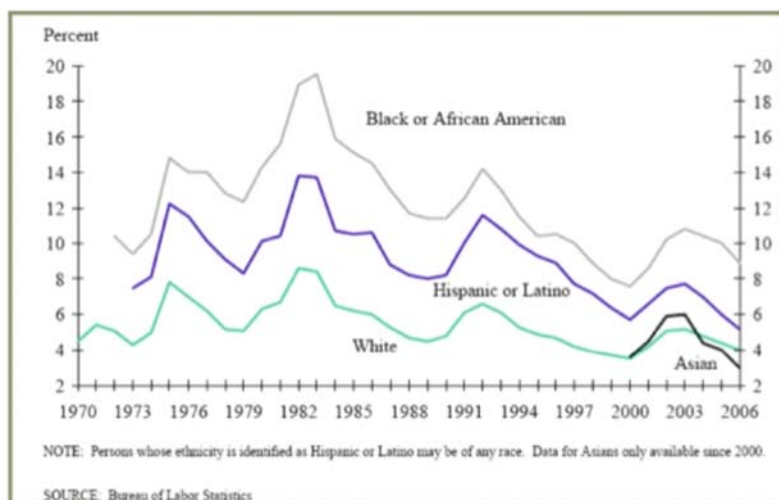
| Unemployment rate in 2007 (Percent) | Education attained              | Median weekly earnings in 2007 (Dollars) |
|-------------------------------------|---------------------------------|--|
| 1.4%                                | Doctoral degree                 | \$1,497                                  |
| 1.3                                 | Professional degree             | 1,427                                    |
| 1.8                                 | Master's degree                 | 1,165                                    |
| 2.2                                 | Bachelor's degree               | 987                                      |
| 3.0                                 | Associate degree                | 740                                      |
| 3.8                                 | Some college, no degree         | 683                                      |
| 4.4                                 | High-school graduate            | 604                                      |
| 7.1                                 | Less than a high school diploma | 428                                      |

Source: Bureau of Labor Statistics, Current Population Survey.

**Panel data** combines cross sectional information and time series. The graph below displays the unemployment rate by race over time. From this graph, we can see how the unemployment rate varies by race in any given year, as well as how these rates change over time.

## Panel Data

This graph displays the unemployment rate by race over time. From this graph, we can see how the unemployment rate varies by race in any given year, as well as how these rates change over time. Source: Bureau of Labor Statistics, <http://www.bls.gov/cps/labor2006/chart4-5.pdf>



## Additional Calculations

Another useful tool in economics is the growth rate. A growth rate measures the percentage change in a variable and is calculated by taking the difference between the new value and the old value then dividing the change by the old value. When multiplied by 100, the rate becomes a percentage. For example, if \$200 is invested in the bank on January 1 and on December 31st the amount has grown to 240 dollars, then the percentage increase is calculated by taking the \$40 increase divided by the original \$200 to find the growth rate of 20 percent.

$$\text{Growth Rate} = (\text{New} - \text{Old}) / \text{Old} \times 100$$

**Example:**

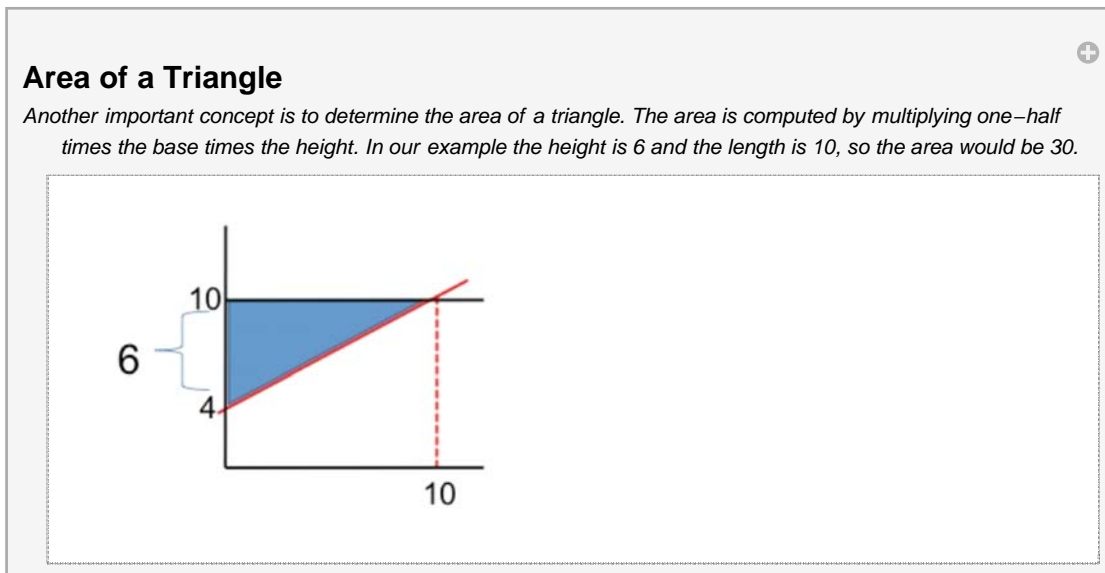
**Invest \$200 on Jan 1 and have \$240 on Dec. 31.**

$$\text{Growth rate} = (\$240 - \$200) / \$200 \times 100 = 20\%$$

Another important concept in economics is that of **average** and **marginal (additional)** amounts. For example, if a car dealer sells 5 cars for a total of \$60,000, the average revenue per car was \$12,000, computed by taking the total revenue divided by the number of cars. The marginal revenue is the additional revenue per car. For example, the additional revenue from selling the fifth car was \$13,000. The marginal revenue is computed by taking the change in total revenue divided by the change in the number of cars. When the fifth car was sold, revenue increased from \$47,000 to \$60,000 or \$13,000.

An individual with \$30,000 of taxable income would have a marginal tax rate of 15 percent. The average tax rate would be the total tax amount paid divided by the taxable income. Since the first \$7,825 is taxed at ten percent, the average tax rate is 13.69 percent. (Source of tax brackets: <http://www.invest-2win.com/Brackets.html>)

Another important concept is to determine the **area of a triangle**. The area is computed by multiplying one-half times the base times the height. In our example the height is 6 and the length is 10, so the area would be 30.



The purpose of math is to help simplify the complex realities of the world. Hopefully, by completing this math review you will find it easier to interpret some of the graphs and charts that you will see throughout this course. If you want a more detailed math review, you may wish to review Chapter 21 of the Rittenburg and Tregarthen text available at the web page below. (Source: <http://www.flatworldknowledge.com/pub/1.0/principles-microeconomics/28243#book-35925>)

## Ponder and Prove - Section 3 - Optimums, Charts & Graphs, and Types of Data

**Section 3 Questions** +

*Instructions: Click on the button that represents the correct answer. After you select an answer, click on the 'Grade My Answer' button.*

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**If GDP is 12,000 in period 1 and 12,500 in period 2, what is the growth rate of GDP from period 1 to period 2?**

4.0%  
 -4.2%  
 4.2%  
 -4.0%

"Results"

## Summary

### Key Terms

Area of a Triangle  
 Bar Chart  
 Cross Sectional Data  
 Dependent Variable  
 Direct Relationship  
 Endogenous Variable  
 Equilibrium  
 Exogenous Variable  
 Graph  
 Growth Rate  
 Horizontal Axis  
 Independent Variable  
 Linear Relationship  
 Marginal  
 Marginal Analysis  
 Maximum  
 Minimum  
 Negative Relationship  
 Optimums  
 Pie Chart  
 Plotting Points

Positive Relationship  
Slope of a Curve  
Solving Equilibrium Algebraically  
Tangent Line  
Time Series Chart  
Values of X  
Values of Y  
Vertical Axis

## Objectives

1. Apply basic mathematical tools to model and solve problems.
2. Recognize math is used as a tool to analyze and display information.
3. Explain the difference between the dependent and independent variables in modeling.
4. Read and analyze data in tabular or graphical format.
5. Graph an equation and numbers from a table.
6. Compute the slope of a curve.
7. Solve equations for unknowns.
8. Compute the rate of change or growth rate.
9. Compute the marginal and average values.
10. Compute the area of a triangle.

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