

Section 3.3 Exercises Part A

Simplify the following.

1. $(3m^2)^5$

2. x^7x^{11}

3. $\frac{f^{12}}{f^5}$

4. t^8t^5

5. 3^{-4}

6. $3x^7 \cdot 4x$

7. $\frac{3z^5}{12z^3}$

8. 17^0

9. $(g^8)^{-2}$

10. $\frac{14d^3}{21d^7}$

11. $(2m^2n^5g^8)^7$

12. $5x^2 \cdot 4x^7$

13. 25 less than 7 times a number is 73. What is the number?

14. Two numbers add to 251 and the second is 41 bigger than the first. What are the two numbers?

Solve.

15. $5p + 12 = 39 - 4p$

16. $5n + 48 = 7n - 2(n - 2)$

17. $15x - 10 = 5(x - 2)$

18. $2(x - 5) - x = 4x - 7$

19. $9m - 3(m - 3) = 15m + 7$

20. $8x - 12x + 3 = 9x + 8x$

21. 45 is what percent of 39?

22. 85 is 24% of what?

23. What is 19% of 3,517?

24. What is 23% of 49?

25. Original Price:
Tax: 5%
Final Price: \$239.40

26. Original Price: \$55.50
Discount: 23%
Final Price:

27. If the population of a town grew 41% up to 7,191. What was the population last year?

28. If the price of an object dropped 35% down to \$11.44, what was the original price?

Solve.

29. $\frac{7}{3}t + 5 = 19$

30. $-\frac{3}{8}(x + 7) = 5 + 3x$

31. $\frac{2}{3}x - 6 = 7 + \frac{1}{2}x$

32. $.3(4x + 7) = 2.5x + 6$

33. $.005x + .045 = .004x$

34. $\frac{x+7}{4} = 4 - \frac{5}{6}x$

Preparation.

35. Simplify the following (so that there are no negative exponents).

$$\frac{5a^{-4}b^{-7}c^{-3}d^{-5}f^{-9}}{7g^{-3}h^{-8}j^{-2}k^{-1}}$$

Answers:

1. $243m^{10}$

2. x^{18}

3. f^7

4. t^{13}

5. $\frac{1}{81}$

6. $12x^8$

7. $\frac{z^2}{4}$

8. 1

9. $\frac{1}{g^{16}}$

10. $\frac{2}{3d^4}$

11. $128m^{14}n^{35}g^{56}$

12. $20x^9$

13. 14

14. 105,146

15. $p = 3$

16. No solution

17. $x = 0$

18. $x = -1$

19. $m = \frac{2}{9}$

20. $x = \frac{1}{7}$

21. 115.4%

22. 354.17

23. 668.23

24. 11.27

25. \$228

26. \$42.74

27. 5,100

28. \$17.60

29. $t = 6$

30. $x = -\frac{61}{27}$

31. $x = 78$

32. $x = -3$

33. $x = -45$

34. $x = \frac{27}{13}$

35.
$$\frac{5g^3h^8j^2k^1}{7a^4b^7c^3d^5f^9}$$

Compounding Quarterly, Monthly, and Daily

So far, you have been compounding interest annually, which means the interest is added once per year. However, you will want to add the interest quarterly, monthly, or daily in some cases. Excel will allow you to make these calculations by adjusting the interest rate and the number of periods to be compounded. **Remember that all interest rates provided in the problems are annual rates. You must adjust them to fit other compounding periods.** The adjusted rate is called the periodic rate. To adjust the periodic rate in Excel, open the FV calculation box and change a 10% annual rate to quarterly, monthly, or daily as follows:

- Quarterly Rate: $.10/4$ Changing the rate to 2.5% or .025
- Monthly Rate: $.10/12$ Changing the rate to .83% or .0083
- Daily Rate: $.10/365$ Changing the rate to .0274% or .000274

Change ten years of compounding to quarterly, monthly, or daily as follows:

- Quarterly Nper: $10*4$ Changing the compounding periods to 40
- Monthly Nper: $10*12$ Changing the compounding periods to 120
- Daily Nper: $10*365$ Changing the compounding periods to 3,650

If you assume you put \$50 into savings and you are comparing savings accounts where the 10% annual interest rate is compounding quarterly, monthly, or daily. You can compare the amount of interest you will earn using Excel as follows:

<u>Quarterly</u>	<u>Monthly</u>	<u>Daily</u>
Rate: $.1/4$ or .025	Rate: $.1/12$ or .00833	Rate: $.1/365$ or .000274
Nper: $10*4$ or 40	Nper: $10*12$ or 120	Nper: $10*365$ or 3650
Pmt: 0	Pmt: 0	Pmt: 0
Pv: -50	Pv: -50	Pv: -50
Future Value = \$134.25	Future Value = \$135.35	Future Value = \$135.90

The more frequently interest is added to your savings and compounded, the more interest you will earn. The above illustration involves a small amount of savings. The more the savings and the more often you add to your savings the more difference it will make when the interest is added and compounded more frequently. The following example illustrates saving \$100 per month for ten years at 10% interest rate compounded monthly versus annually.

<u>Annually</u>	<u>Monthly</u>
Rate: .1 or 10%	Rate: $.1/12$ or .00833
Nper: 10	Nper: $10*12$
Pmt: -1200	Pmt: -100
Pv: 0	Pv: 0
Future Value = \$19,124.91	Future Value = \$20,484.50

Savings Plan Formula for a lump sum

$$A = P \left(1 + \frac{r}{n} \right)^{nY}$$

Savings Plan Formula with payment

$$A = PMT \left(\frac{\left(1 + \frac{r}{n} \right)^{nY} - 1}{\frac{r}{n}} \right)$$

Thus we have the monster formula for a Savings Plan that begins with a balance and then is added to by a payment:

$$A = P \left(1 + \frac{r}{n} \right)^{nY} + PMT \left(\frac{\left(1 + \frac{r}{n} \right)^{nY} - 1}{\frac{r}{n}} \right)$$

Spreadsheets normally have this formula built into their functions. It is known as Future Value (FV), so you won't need to use this one if you learn the spreadsheet well.

Loan Payment Formula

$$PMT = P \left(\frac{\frac{r}{n}}{1 - \left(1 + \frac{r}{n} \right)^{-nY}} \right)$$

Spreadsheets also normally have this formula built into their functions. It is known as Payment (PMT).

Final note using a spreadsheet: The formulas are built so that money going out from you is negative and money coming in to you is positive. When you are entering Savings into the spreadsheet, the payment and Principal (Present Value) will be negative. However, for a loan, the payment will be negative but the Principal (Present Value) will be positive, because it represents money coming to you.

A = Final Amount
PMT = monthly payment
P = Principal amount (beginning balance)
r = annual interest rate
n = number of compounding per year
Y = number of years

So,
 $\frac{r}{n}$ = periodic interest rate (rate used in spreadsheet)

nY = number of periods (nper)

Calculating Payments, Interest Rates, and Number of Periods

Excel will help you calculate the payment you will need to make on a loan. It will calculate the interest rate you would need to earn on your savings to realize a certain future balance. The number of periods it will take to have your savings grow to a certain future balance can also be determined.

Monthly Payment Calculation

If you wanted to buy a car that costs \$15,000 and you can get a loan at 6% interest for four years, you can determine the monthly payments using the PMT Excel function as follows:

Rate: $.06/12$ or $.005$ (monthly interest)
 Nper: $4*12$ or 48 (months)
 Pv: -15000
 Fv: 0
 Monthly Payment = \$352.28

When you have paid the monthly payment for forty-eight months you will own the car and the future value of the loan is zero because the loan is paid off.

Benefits Versus Bondage

You can see how hard your savings will work for you given an interest rate and enough time. However, interest works against you when you borrow money. The benefits may seem great at the moment but the financial bondage is terrible. By calculating the interest you would pay on a loan to borrow a car and the interest you would earn by saving to be able to pay cash for the car, we can determine the financial advantage of collecting interest rather than paying interest.

Interest Paid on a Car Loan

You calculate the amount of interest you would pay on a four year car loan of \$15,000 at 6% annual interest using the Excel Pmt function as follows:

Rate: $.06/12$
 Nper: $4*12$
 Pv: -15000
 Fv: 0
 Monthly Payment = \$352.28
 Total Payment = $352.28*48$ (Payments) = \$16,909.22
 Interest Paid = $16,909.22$ (Paid) - $15,000$ (Borrowed) = \$1,909.22

TIP: You can have Excel calculate this for you by entering the Pmt function to calculate the monthly payment and then, on the formula bar at the top of the Excel sheet, multiply by 48 payments and subtract the \$15,000 you borrowed. The formula will be as follows:

$$=PMT(0.06/12,4*12,-15000,0)*48-15000$$

You can also double click on the cell with the Pmt calculation in it and the formula will appear in the cell. Now you can multiply by 48 payments and subtract 15000 and enter this formula in the cell. The cell will have the answer and the formula will be in the formula bar.

Interest Collected on Your Savings

The interest you will earn on your savings of \$350.00 per month earning 6% annual interest for 39 months (the number of months we calculated above would be required to accumulate \$15,000 in savings) is calculated using the FV function in Excel as follows:

Rate: .06/12
Nper: 39
Pmt: -350
Pv: 0

$$FV = \$15,030.44$$

$$\text{Amount Deposited in Savings} = \$350 * 39 \text{ (deposits)} = \$13,650.00$$

$$\text{Interest Earned on Savings} = \$15,030.44 - \$13,650.00 = \$1,380.44$$

Again, you can double click on the cell containing the FV calculation and subtract $350 * 39$ and enter this formula giving you the amount of interest earned. You can make the same adjustment to the formula in the formula bar. The resulting formula is as follows:

$$=FV(0.06/12,39,-350)-350*39$$

Total Savings From Saving Versus Borrowing

Here is how you benefited by saving and paying cash for the car rather than borrowing the money to buy the car:

Interest Earned	\$1,380.44
Interest Not Paid	<u>\$1,909.22</u>
Financial Advantage	\$3,289.66

You are wealthier by \$3,289.66 because you collected interest rather than paying interest. This practice will make a major difference in your financial well being throughout your life. If you put the money you save by paying cash for major purchases to work for you by investing it for your retirement you will add greatly to your independent wealth. You

can estimate that using the FV function in Excel as follows assuming a 6% return on your investment for 30 years:

Rate: .06
Nper 30
Pv: -3289.66
FV = \$18,894.13

This addition to your wealth along with the other additions resulting from saving rather than borrowing will make a major impact on your ultimate wealth.

TIP: In all of the Excel functions you will be using, you only need three entries or factors to calculate the fourth factor you are after. Notice that there are only three entries in each of the above Excel functions. You can leave blank any factor not needed and Excel will assume it is zero.